

# Implementing Effective Math Interventions for Fluency & Problem Solving

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## Does Math Phobia Impact Learning?

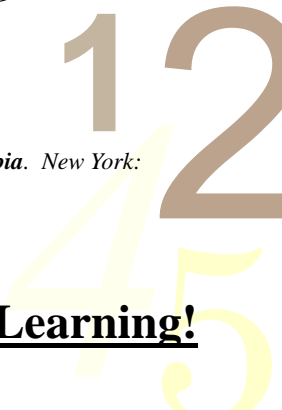
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*Math is right up there with  
snakes, public speaking, and  
heights.*

Burns, M. (1998). *Math: Facing an American Phobia*. New York:  
Math Solutions Publications.



**Math Perceptions Can Hinder Learning!**



## Math Interventions Are Critical

“Math is highly proceduralized and continually builds on previous knowledge for successful learning. Hence, *early deficits* have **enduring** and **devastating** effects on later learning, as indicated in *The Head Start Path to Positive Child Outcomes*.”

(U.S. Department of Health and Human Sciences, 2001; National Mathematics Advisory Panel, 2008; National Council of Teachers of Mathematics, 2000; and U.S. Department of Education, 2003)

Source: Amanda VanDerHeyden, Ph.D. from Education Research and Consulting, Inc.

## Math Developmental Timeline

- **Preschool Years:**
  - Development of the “*number sense*” needed to process and manipulate numerical information
- **Primary School:**
  - Understanding of *foundation number concepts and relationships* among numbers
  - Internalizing the number line leading to mental arithmetic
- **Elementary School:**
  - Mastering *common math operations with fluency*
  - Applying and refining skills and strategies necessary to *solve real life math problems*
- **Middle School:**
  - Effective and efficient use of problem solving skills and strategies in school, home, and community

Source: *Math Problem Solving for Primary Elementary Students with Disabilities* by Montague

## How Many Students Do You Suspect Are Performing According to the Developmental Timeline?

- **Greater than 1/3 of students assessed at grades 4, 8, and 12 failed to reach basic levels of proficiency**
  - *National Assessment of Educational Progress (NAEP)*

## Our Focus Today: Most Common Math Referrals

- **Two Types of Deficits** (*Shapiro, 1996*)
  - **Mastering Computational Skills**
    - Basic operations and fluency
  - **Applying Math Concepts**
    - Money, measurement, time, and word problems

## Math Intervention Strategies

- **Concrete-Representational-Abstract**
  - Computation & Applying Concepts
- **Cover, Copy, Compare**
  - Computation
- **Computer Assisted Instruction**
  - Computation & Applying Concepts
- **Cognitive Strategy Instruction**
  - Computation & Applying Concepts
- **Schema Based Instruction**
  - Applying Math Concepts
- **Peer Assisted Learning**
  - Computation & Applying Concepts
- **Think Aloud**
  - Applying Math Concepts

## Best Practice in Teaching Strategies

1. Pretest- *measure skills*
2. Describe- *explain steps and how to use*
3. Model- *demonstrate with think aloud*
4. Verbal Practice- *memorize steps*
5. Controlled Practice- *ensure mastery with simplified materials & controlled setting*
6. Grade-Appropriate Practice- *ensure mastery with grade appropriate material*
7. Post test- *measure skill obtainment*
8. Generalization- *apply strategies in other settings*

(Schumaker & Deshler, 1992)

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# Math Computational Skills

## *Basic Operations & Fluency*



- **Concrete**
- **Representational**
- **Abstract**

Focus: Understand  
underlying math concepts  
before learning "the rules"



## Concrete-Representational-Abstract

- **Three Part Instructional Method**
  - **Concrete:** Concrete representations of the problem with objects...DOING
  - **Representational:** Pictorial representations of the amounts...SEEING
  - **Abstract:** Numeric representations of amounts...SYMBOLIC
- Each part builds on the previous instruction to
  - promote student learning
  - promote retention
  - address conceptual knowledge

## Math Concepts Suited for C-R-A

- |                          |                 |
|--------------------------|-----------------|
| • Early number relations | • Money         |
| • Place value            | • Percentage    |
| • Computation            | • Number bases  |
| • Fractions              | • Word problems |
| • Decimals               | • Probability   |
| • Measurement            | • Statistics    |
| • Geometry               |                 |

Source: The Access Center

## Concrete Example

- Sarah wanted a comic book for \$4.50. On Friday, her dad gave her \$10.00 for mowing the lawn. Sarah paid her brother the \$2.00 she owed him. Then, her sister asked to borrow \$3.00. Does Sarah have enough money to buy the book she wanted?

\$ \$    \$ \$ \$

$$\$10.00 - \$2.00 = \$8.00$$

\$ \$ \$    \$ \$

$$\$8.00 - \$3.00 = \underline{\$5.00}$$

## Concrete Lesson In Action...

- 3 green bears are joined by 2 red bears in the lake...
  - Plastic bears on chart paper
  - Plastic bears in container of water

- SOURCE:

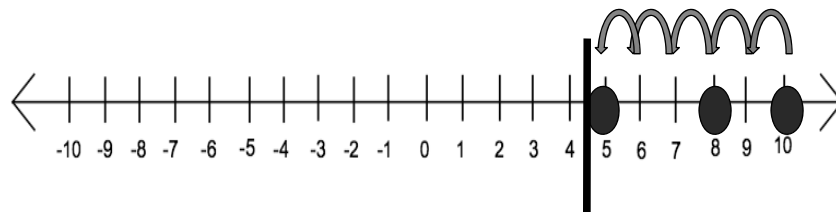
<http://fcit.usf.edu/mathvids/videos/videos.html>

## Representational Example

Sarah wanted a comic book for \$4.50. On Friday, her dad gave her \$10.00 for mowing the lawn. Sarah paid her brother the \$2.00 she owed him. Then, her sister asked to borrow \$3.00. Does Sarah have enough money to buy the book she wanted?

$$\mathbf{\$10.00 - \$2.00 = \$8.00}$$

$$\mathbf{\$8.00 - \$3.00 = \underline{\$5.00}}$$



## Representational Lesson In Action...

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- 12 cookies are shared by 2 people...

- Tally marks

- SOURCE:

<http://fcit.usf.edu/mathvids/videos/videos.html>

12  
45

## Abstract Example

– Sarah wanted a comic book for \$4.50. On Friday, her dad gave her \$10.00 for mowing the lawn. Sarah paid her brother the \$2.00 she owed him. Then, her sister asked to borrow \$3.00. Does Sarah have enough money to buy the book she wanted?

– Numerical Symbols

- $\$10.00 - \$2.00 = \$8.00$
- $\$8.00 - \$3.00 = \$5.00$
- \$4.50 is less than \$5.00
- Sarah has enough money to buy the comic book.

## Abstract Example

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- **SOURCE:**  
<http://fcit.usf.edu/mathvids/videos/videos.html>

## Implementing CRA Strategy: The Concrete Stage

- **Pre-Assess:**

- Assess student's ability to solve math word problems with concrete objects

- **Describe and Model:**

- Initial Instruction: Describe & model it using concrete objects (concrete level of understanding)
- Abstract numbers/symbols should be used with the concrete materials and representational drawings to promote association

- **Verbal Practice:**

- Memorize steps to using the concrete objects (based on problem type...decimals, computation, Geometry)

## Implementing CRA Strategy: The Concrete Stage

- **Controlled Practice:**

- Provide students many practice opportunities using concrete objects.
- Work with whole numbers under 10 or keep it at 1 or 2 step problems

- **Grade-Appropriate Practice:**

- Use grade appropriate material with the concrete objects

- **Post-Test:**

- Measure Skill Attainment with concrete objects
- Can they use this strategy effectively and independently? If so, move on to the next stage. If not, go back through the instructional steps systematically.

## Implementing CRA Strategy: The Representation Stage

- **Pre-Assess:**
  - Assess student's ability to solve math word problems with representations
- **Describe and Model:**
  - Describe & model how to perform the skill by drawing or with pictures that represent concrete objects (representational level of understanding)
    - Tally marks, pictures, number line, etc.
  - Abstract numbers/symbols should be used with the concrete materials and representational drawings to promote association
- **Verbal Practice:**
  - Memorize steps to using the representational level of understanding (based on problem type...money, fractions, etc.)

## Implementing CRA Strategy: The Representation Stage

- **Controlled Practice:**
  - Provide many practice opportunities for students to draw their solutions or use pictures to problem-solve.
  - Work with whole numbers under 10 or keep it at 1 or 2 step problems
- **Grade-Appropriate Practice:**
  - Use grade appropriate material with the representations
- **Post-Test:**
  - Measure Skill Attainment with representations
  - Can they use this strategy effectively and independently? If so, move on to the next stage. If not, go back through the instructional steps systematically.

## Implementing CRA Strategy: The Abstract Stage

- **Pre-Assess:**
  - Assess student’s ability to solve math word problems in an abstract manner
- **Describe and Model:**
  - Describe and model how to perform the skill using only numbers and math symbols (abstract level of understanding)
- **Verbal Practice:**
  - Memorize steps to using the representational level of understanding (based on problem type...percentage, probability, statistics, etc.)

## Implementing CRA Strategy: The Abstract Stage

- **Controlled Practice:**
  - Provide many opportunities for students to practice performing the skill using only numbers and symbols.
  - Work with whole numbers under 10 or keep it at 1 or 2 step problems
- **Grade-Appropriate Practice:**
  - Use grade appropriate material with the abstract stage
- **Post-Test:**
  - Measure Skill Attainment with abstract stage
  - Can they use this strategy effectively and independently? If so, move on to the next stage. If not, go back through the instructional steps systematically.
- Upon mastery at the *abstract* level of understanding: Ensure maintenance by providing periodic practice opportunities for the math skills.

## C-R-A Case Study

- Haley and her friends are playing hot potato. Haley passes the ball to Sarah who passes the ball to Payton who passes the ball to Wesley who passes the ball back to Haley. Who will have the ball after the 9<sup>th</sup> pass?

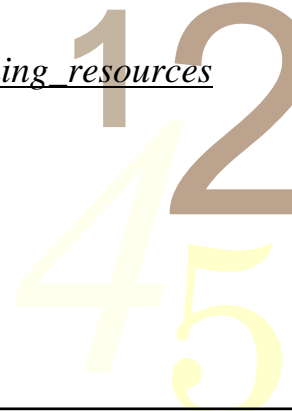
## C-R-A Case Study

- What are the specific steps to solving this type of problem?
- What manipulatives could be used at the concrete stage?
- What representations could be made?
- What could be done to simplify this problem for controlled practice?

## CRA Resources

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- **Video Instructional Development Sources:**  
<http://coe.jmu.edu/mathvids2/strategies/cra.html>
- **The Access Center:**  
[http://www.k8accesscenter.org/training\\_resources/CRA\\_Instructional\\_Approach.asp](http://www.k8accesscenter.org/training_resources/CRA_Instructional_Approach.asp)



## Cover, Copy, & Compare: Increasing Math Fluency

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## **Math Fluency:**

### **Cover, Copy, & Compare Overview**

- Strategy for improving accuracy and speed with basic mathematics facts
- Requires minimal teacher time & student training
- Individual, small group, or class-wide implementation
- Explicitly teaches students how to:
  - *Self-instruct*
  - *Self-question*
  - *Self-monitor*

### **Cover, Copy, & Compare Materials:**

- Training sheets of 10 math problems, with problems and answers listed down the left side of the paper, one per student, one to three sets per session
- Assessment sheets with the same math problems listed down the left side, without answers
  - <http://www.interventioncentral.org/htmldocs/tools/mathprobe/addsing.php>
- 3” by 5” index cards, one per student

## Cover, Copy, & Compare: *Pre-Assessment Option*

- Assess students' rate and accuracy of math computation with Curriculum-Based Mathematics Probe for 5-10 days
  - **Accuracy:** Count the number of correct digits on math problem worksheets.

$$\begin{array}{r}
 52 \\
 3 \text{ CORRECT DIGITS} \quad \times \quad \underline{2} \\
 \hline
 104
 \end{array}$$

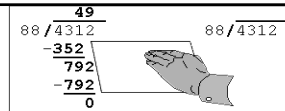
## Cover, Copy, & Compare: *Pre-Assessment Option*

- Assess students' rate and accuracy of math computation with Curriculum-Based Mathematics Probe for 5-10 days
  - **Rate:** Count the number of correct digits, multiply by 60, and divide by the number of seconds allotted to the task to get digits correct per minute
  - *Payton completed 12 correct digits in a five minute period of time.*
  - *What was his computation rate?*
  - $12 \times 60 / 60 \times 5$
  - $720 / 300 =$  **2.4 correct digits per minute**

## Implementing Cover, Copy & Compare

- **Describe Strategy Steps:**
  - Read the problem and the answer
  - Cover the problem and the answer
  - Copy the problem and the answer from memory
  - Compare your answer with the original answer
  - If incorrect, repeat the process
  - If correct, repeat steps with a new problem

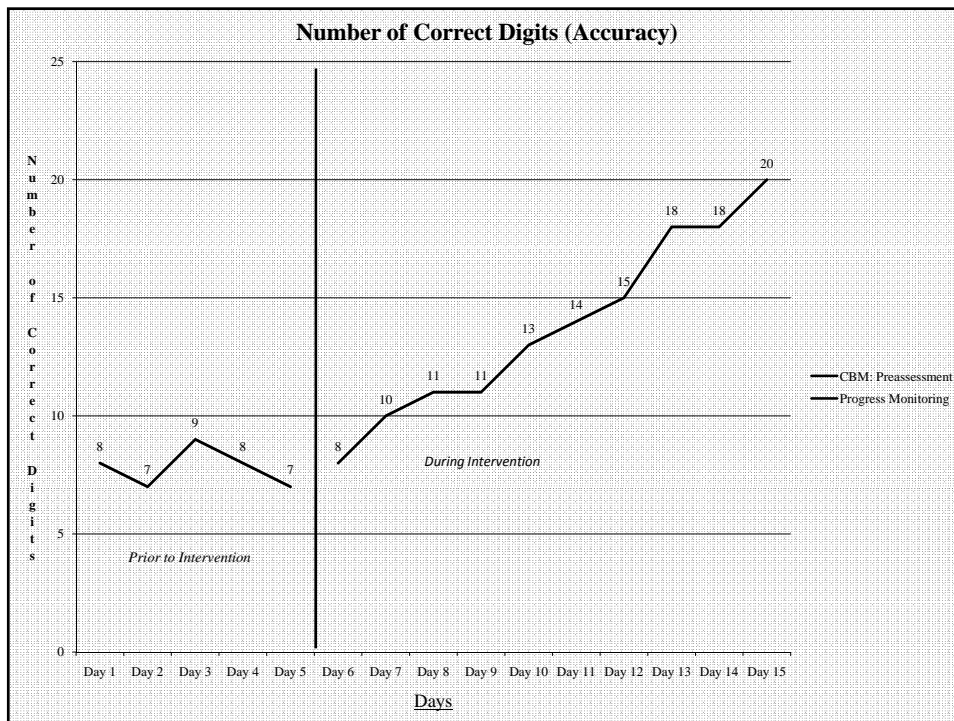
## Implementing Cover, Copy & Compare



- **Model:**
  - Silently read the first problem and the answer on the left side of the paper.
  - Cover that problem and answer with an index card.
  - Write the problem and answer from memory on the right side of the page.
  - Uncover the problem and answer on the left side to check the written response.
  - Evaluate the response.
  - If the problem and answer are written incorrectly, repeat the procedure with that item before proceeding to the next item.
  - Repeat this procedure with the rest of the problems on the sheet.

# Implementing Cover, Copy & Compare

- **Verbal Practice:**
  - Have students memorize and recite steps...Read, Cover, Copy, Compare
- **Controlled Practice:**
  - Following the demonstration, have students complete one or more training sheets with corrective feedback as needed.
- **Grade-Appropriate Practice:**
  - Daily or several times a week, provide students with sets of training sheets and have them follow the *Cover, Copy, and Compare* procedure.
- **Post Test:**
  - Once or twice a week, administer the assessment sheets that correspond to the training sheets. If desired, time these assessment sessions.
  - When students reach mastery level on one set of problems, provide them with another set. *Mastery level is defined as 90% or better accuracy and/or 40 digits correct per minute.*



## Cover, Copy, and Compare: Increasing Math Fluency

### • Sources:

- Rathovan, Natalie (1999). Effective School Interventions. Guilford Press: New York, NY.
- Lee, M.J., & Tingstrom, D.H. (1994). A group math intervention: The modification of cover, copy, and compare for group application. Psychology in the Schools, 31, 133-145.
- Skinner, C.H., Turco, T.L., Beatty, K.L., & Rasavage, C. (1989). Cover, copy, and compare: A method for increasing multiplication performance. School Psychology Review, 18, 412-420.
- <http://www.jimwrightonline.com/pdfdocs/ccc.pdf>

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## Computer Assisted Instruction

1 2  
4 5

## Computer Assisted Instruction (CAI)

- **Stand-alone computer learning OR Supplemental to teacher directed instruction**
  - *Drill and practice*
  - *Tutorials*
  - *Simulation activities*
- **Research base**
  - Achievement effect of conventional instruction **and** CAI is superior to those obtained with traditional instruction alone
    - “Well-designed and implemented D&P (drill and practice) or tutorial CAI, used as a supplement to traditional instruction, produces an educationally significant improvement in students’ final examination achievement” (Stennett, 1985, p.7)
    - “While both traditional and computer-based delivery systems have valuable roles in supporting instruction, they are of greatest value when complementing one another” (Dalton & Hannafin, 1988, p. 32)
  - Learning rate & retention higher for CAI than conventional instruction alone

## Student Comments on CAI

- **Computers:**
  - Never get tired
  - Never get frustrated or angry
  - Allow students to work privately
  - Never forget to correct or praise
  - Are fun and entertaining
  - Individualize learning
  - Are self-paced
  - Do not embarrass students who make mistakes
  - Make it possible to experiment with different options
  - Give immediate feedback
- Are more objective than teacher
- Free teacher to work with individuals
- Are impartial to race or ethnicity
- Are great motivators
- Give a sense of control over learning
- Are excellent for drill and practice
- Call for using sight, hearing, and touch
- Teach in small increments
- Build proficiency in computer use (invaluable for later in life)
- Eliminate the drudgery of certain learning activities
- Work rapidly to match rate of human thought

Adapted from NWREL's School Improvement Research Series (May 1991)

## Computer Assisted Instruction for Math

- <http://www.aplusmath.com/>
- <http://www.coolmath4kids.com/>
- [http://www.edinformatics.com/kids\\_teens/kt\\_math.htm](http://www.edinformatics.com/kids_teens/kt_math.htm)
- <http://www.figurethis.org/index.html>
- <http://www.kcw.org/reprek6.htm>
- [http://www.ldonline.org/ld\\_indepth/technology/babbitt\\_math\\_tips.html](http://www.ldonline.org/ld_indepth/technology/babbitt_math_tips.html)
- <http://mathforum.org/arithmetic/arith.software.html>
- <http://matti.usu.edu/nlvm/nav/index.html>
- <http://www.netn.net/14113.htm>
- [http://www.montgomeryschoolsmd.org/schools/thurgoodmarshalles/java-applets/mathflash/mathflash\\_small.html](http://www.montgomeryschoolsmd.org/schools/thurgoodmarshalles/java-applets/mathflash/mathflash_small.html)

Source: *The Access Center: Improving Outcomes for All Students K-8;*

*\*\*For additional resources see Using RTI for School Improvement: Raising Every Student's Achievement Scores by Shores & Chester*

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## Applying Math Concepts: Problem Solving

*“Many students in kindergarten through grade 3, especially students with learning disabilities, have difficulty learning how to solve math word problems because they often do not have the necessary conceptual bases....*

*...Therefore they **need explicit instruction** in mathematical problem-solving skills and strategies to solve problems in their math textbooks and in their daily lives.”*

Source: *Math Problem Solving for Primary Elementary Students with Disabilities* by Montague

## Barriers to Solving Math Problems

- Reading skill
- Comprehension
- Lack of proficiency with math operations
- Lack of fluency with math facts

## Problem Solving Skills Include:

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- Understanding specialized math vocabulary
- Identifying relevant math operations
- Ignoring unnecessary information
- Translating word problems from text to numeric representations (digits and math symbols)
- Solving

[www.interventioncentral.org](http://www.interventioncentral.org)

## Problem Solving Instructional Steps

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- Focus on:
  - Reading
  - Paraphrasing
  - Visualizing
- Teacher assists students in visually representing the problem with manipulatives (CONCRETE)
- Teacher assists students in creating a representation with paper and pencil (REPRESENTATIONAL)
- Teacher assists students in making a symbolic representation of the problem (ABSTRACT)

## Peer Assisted Learning

- Structured practice in which students solve problems in pairs
- Two roles that rotate
  - Coach & player
  - Thinker & listener
- Students **MUST** be trained in tutoring procedures
  - Prompts or cue sheets can be used for appropriate corrective feedback
- Teacher actively models, guides, and supports

## Research on Peer Tutoring

- *Nearly two decades of research have supported the strategy as effective. Two year and 12 year classroom follow-up studies indicated that classwide peer tutoring led to faster, more effective student learning outcomes than did teacher-mediated instruction... Further, at-risk first-graders who received a classwide peer tutoring intervention in spelling, math, and reading demonstrated long-term benefits, compared to students in control groups, with fewer referrals to special services and lower dropout rate in the 11<sup>th</sup> grade.*
- Source: Teaching Exceptional Children: Classwide Peer Tutoring at Work (Nov/Dec 2001)

## Peer Assisted Learning Strategies (PALS)

- Source: <http://kc.vanderbilt.edu/pals/>
  - PALS Outreach  
Vanderbilt University  
Peabody Box 328  
110 Magnolia Circle  
Nashville, TN 37203-5701  
615-343-4782

## Thinking Out Loud

**Source:** *What Successful Math Teachers Do, Grades  
6-12: 79 Research Based Strategies for the  
Standards Based Classroom*

## Thinking Out Loud Overview

- Forces students to pay attention to their thinking and problem solving
- Increases awareness of the information and how to solve the problem
- Increases systematic thinking

## National Council of Teachers of Mathematics *Communication* Standards

- Organize and consolidate their mathematical thinking through communication;
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- Analyze and evaluate the mathematical thinking and strategies of others;
- Use the language of mathematics to express mathematical ideas precisely.

## Implementing Thinking Out Loud

1. **Pretest-** *Assess student need for a problem solving strategy*
2. **Describe-** *explain steps and how to use*
  1. *In pairs, students take turns thinking out loud while solving problems*
  2. *One student is the “thinker” and one is the “listener”*
  3. *Role of the “thinker”: express the methods used to solve the problem*
  4. *Role of the “listener”: actively listen and offer own strategy when roles flip*
3. **Model-** *demonstrate the strategy with acceptable responses...cue cards*

## Implementing Thinking Out Loud

4. **Verbal Practice-** *memorize steps (next slide)*
5. **Controlled Practice-** *ensure mastery with simplified materials & controlled setting*
6. **Grade-Appropriate Practice-** *ensure mastery with grade appropriate material*
7. **Post test-** *measure specific skill obtainment*
8. **Generalization-** *apply strategies in other settings*

*(Schumaker & Deshler, 1992)*

## Thinking Out Loud Strategy Steps

- Listen for details as the “thinker” explains how to solve the problem
- Think to yourself, “Does that make sense?” and “How did I solve it?”
- Respond respectfully
- Switch roles

## Cognitive Strategy Instruction:

*One of the most powerful interventions for students with learning disabilities*

*(Swanson, 1999)*

- Resource:  
University of Nebraska-Lincoln: Cognitive Strategy Instruction  
– <http://www.unl.edu/csi/>

## Cognitive Strategy Instruction

- **Guides** that serve to support the learner as they develop internal procedures to perform higher level operations
- **Focus:** *Process* information and become independent learners
- **Strategies include:**
  - Visualization, verbalization, making associations, chunking, questioning, scanning, underlining, accessing cues, and mnemonics

<http://www.unl.edu/csi/> & The Access Center

## CSI Math Examples

- Self Monitoring Strategies
- Rules to Lower the Amount of Memorization in Math
- STAR: A Number Writing Strategy
- SLOBS & LAMBS: Regrouping and Borrowing Strategy for Addition and Subtraction
- Multiplication Attach Strategy
- Draw & Fast Draw: Solving Multiplication Facts
  - **Resource:**  
University of Nebraska-Lincoln: Cognitive Strategy Instruction
  - <http://www.unl.edu/csi/>

## ORDER:

### *A Cognitive Strategy for Order of Operations*

- SOURCE:  
<http://fcit.usf.edu/mathvids/videos/videos.html>

### Steps for Cognitive Strategy Instruction

- Teach prerequisite skills (based on pretest) and activate prior knowledge
- Describe strategy to students with prompts/cues
- Teach the strategy in small steps
- Model the strategy with “think alouds”
- Students verbally rehearse to memorize steps and uses checklists to assist
- Guided practice with corrective feedback to support the strategy
- Independent practice
- Generalization and self monitoring for mastery

(Harris & Graham, 1996; Lloyd, Kameanui, and Chard, 1997; Montague, 1992, 1995, 1997)

## Research on Cognitive Strategy Instruction

- Case, Harris & Graham (1992) – self-regulated strategy on problem solving
- Hutchinson (1993) representing and solving word problems
- Jitendra & Hoff (1995) – **schema-based instruction**, distinguish type of problem
- Jitendra & Xin (1997) meta-analysis
- Maccini & Hughes (2000) problem-solving strategy
- Montague (1992) cognitive and metacognitive strategy
- Montague, Applegate & Marquard (1993) cognitive and metacognitive strategy
- Montague, Warger & Morgan (2000) Solve It!
- Woodward, Monroe, & Baxter (2001) classwide performance tasks

## Schema Based Instruction

**Resource:** Solving Math Word Problems: Teaching Students with Learning Disabilities Using Schema-Based Instruction by Asha Jitendra

## Schema Based Instruction

- Is a form of Cognitive Strategy Instruction
- Four step process (FOPS):
  - Find the problem type
    - *Change*
    - *Group*
    - *Compare*
  - Organize the information using a diagram
  - Plan to solve the problem
  - Solve

## Schema Based Instruction Problem Types

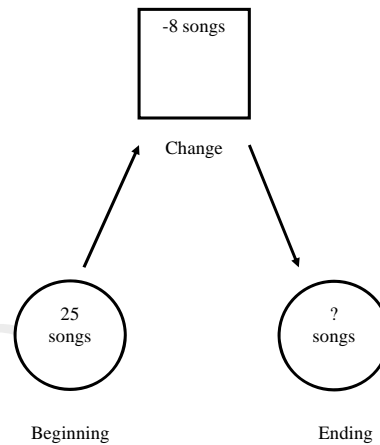
- Students first categorize word problems as being a
  - *Change* problem
    - Tells a story
    - Involves addition or subtraction
      - *Chris downloaded 25 of his favorite songs to his new iPod. His mom made him remove 8 of these. How many songs does he now have on his iPod?*
  - *Group* problem
    - Distinct groups combine to form a new group or set
      - *Haley bought 6 new outfits with her birthday money. Two of the outfits were from Target. How many outfits came from Old Navy?*
  - *Compare* problem
    - Relationships between two different number
      - *Payton's team scored 16 points during the basketball game. Anna's team scored 8 points less than Payton's. How many points did Anna's team score in the basketball game?*

## Change Problem

Chris downloaded 25 of his favorite songs to his new iPod.  
His mom made him remove 8 of the songs. How many songs does he now have on his iPod?

Four step process:

- Find the problem type
- Organize the information using a diagram
- Plan to solve the problem
- Solve



Math Sentence:

$$\begin{array}{r} 25 \\ -8 \\ \hline 17 \\ \text{songs} \end{array}$$

## Change Problem Checklist

### 1. Find the problem type

- Did I read and retell the problem?
- Did I ask if it is a *change* problem? (look for the beginning, change, and ending)

### 2. Organize the information using the change diagram

- Did I underline the label that describes the beginning, change, and ending?
- Did I label the amounts in the diagram?
- Did I underline the important information, circle the numbers, and write the numbers in the diagram?
- Did I write a “?” for what must be solved?

## Change Problem Checklist

3. Plan to solve the problem
  - Do I add or subtract?
  - Did I write the math sentence?
4. Solve the problem
  - Did I solve the math sentence?
  - Did I write the complete answer?
  - Did I check if the answer makes sense?

## Change Problem

*Chris downloaded 25 of his favorite songs to his new iPod. His mom made him remove 8 of the songs. How many songs does he now have on his iPod?*

Four step process:

- Find the problem type
- Organize the information using a diagram
- Plan to solve the problem
- Solve

```

graph TD
    A((25 songs)) --> B[Change]
    B --> C((? songs))
            
```

**Math Sentence:**

$$\begin{array}{r} 25 \\ - 8 \\ \hline 17 \\ \text{songs} \end{array}$$

## Group Problem

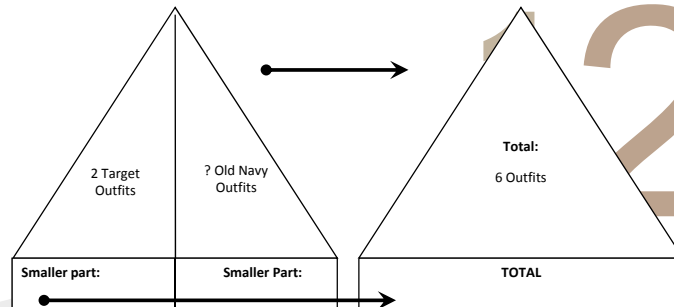
Haley bought 6 new outfits with her birthday money. Two of the outfits were from Target. How many outfits came from Old Navy?

Four step process:

- Find the problem type
- Organize the information using a diagram
- Plan to solve the problem
- Solve

**Math Sentence:**

$$6 - 2 = 4 \text{ outfits}$$



## Compare Problem

Payton's team scored 16 points during the basketball game. Anna's team scored 5 points less than Payton's. How many points did Anna's team score in the basketball game?

Four step process:

- Find the problem type
- Organize the information using a diagram
- Plan to solve the problem
- Solve

**Greater:**

Payton : 16 points

**Lesser:**

? Anna's Total Points

**Difference:**

Anna: 5 points less

**Math Sentence:**

$$16 - 5 = 11 \text{ points}$$

## Implementing Schema Based Instruction

1. **Pretest-** Determine if there is a need for a problem solving strategy
2. **Describe-** FOPS (see checklist) & Problem types
3. **Model-** Demonstrate each step of FOPS with think aloud
4. **Verbal Practice-** Student verbally states steps (FOPS)
5. **Controlled Practice-** Ensure mastery with simplified materials & controlled setting- master one type of problem first
6. **Grade-Appropriate Practice-** Ensure mastery with grade appropriate material
7. **Post test-** Measure schema based skill obtainment
8. **Generalization-** Apply strategies in other settings

(Schumaker & Deshler, 1992)

## Research on Schema Based Instruction

- Students who learn to identify three different kinds of word problems and their strategies do better on math tests than students who learn *only* one general-purpose model – *Journal of Educational Psychology* (February) vol. 99, pages 115-127
- Stopping to categorize a word problem before picking a plan to solve it may be especially effective for low-achieving students – *Asha Jitendra, PhD from Lehigh University*

## Schema Based Instruction Resources:

- An Exploratory Study of Schema-Based Word-Problem-Solving Instruction for Middle School Students with Learning Disabilities: An Emphasis on Conceptual and Procedural Understanding
  - <http://www.ldonline.org/article/5678>
- Teaching Students Math Problem-Solving Through Graphic Representations
  - [www.teachingld.org/pdf/teaching\\_how-tos/journal\\_articles/Article\\_5.pdf](http://www.teachingld.org/pdf/teaching_how-tos/journal_articles/Article_5.pdf)

## Problem Solving Sources:

- Problem Solving and Comprehension by Whimbey & Lochhead
- Effects of Overt, Controlled Verbalization and Goal-Specific Search on Acquisition of Procedural Knowledge in Problem Solving by Zook & DiVesta
- What Successful Math Teachers Do by Posamentier & Jaye

## Effective Math Instruction Includes

- A system for monitoring student learning
- Adjustment of instruction to ensure learning
- A well sequenced program of instruction that logically builds on existing skills and returns to previously mastered skills to ensure maintenance
- Modeling of correct responses
- Substantial opportunity to practice newly learned skills with support (such as corrective feedback)
- Independent practice

*Source: Amanda VanDerHeyden, Ph.D. from Education Research and Consulting, Inc.*

## Concluding Thoughts

According to the National Research Council  
(1989):

Mathematics is the key to opportunity. No longer just the language of science, mathematics now contributes in direct and fundamental ways to business, finance, health, and defense. For students, it opens doors to careers. For citizens, it enables informed decisions. For nations, it provides knowledge to compete in a technological economy. To participate fully in the world of the future, America must tap the power of mathematics (p.1).

*Source: The Access Center*

## Resources:

- **What Successful Math Teachers Do, Grades 6-12: 79 Research-Based Strategies for the Standards-Based Classroom** by Posamentier and Jaye
- **Effective School Intervention: Strategies for Enhancing Academic Achievement and Social Competence** by Rathvon
- **Teaching Mathematics to Middle School Students with Learning Difficulties** by Montague and Jitendra
- **Using RTI for School Improvement: Raising Every Student's Achievement Scores** by Shores and Chester

## Web Resources:

- What Works Clearinghouse:  
– <http://ies.ed.gov/ncee/wwc/>
- University of Nebraska-Lincoln: Cognitive Strategy Instruction  
– <http://www.unl.edu/csi/>
- John Hopkins University: Best Evidence Encyclopedia  
– [www.bestevidence.org](http://www.bestevidence.org)
- RTI Action Network  
– <http://www.rtinetwork.org/>
- National Center on Response to Intervention  
– <http://www.rti4success.org/>
- Intervention Central  
– [www.interventioncentral.org](http://www.interventioncentral.org)